# F-35 O-Ring Production Functions versus Mosaic Warfare

## Some Simple Mathematics

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## Introduction

On 28 January 1986, the space shuttle *Challenger* broke apart 73 seconds into its flight, claiming the lives of all seven astronauts aboard. The Presidential Commission on the Space Shuttle Challenger Accident, known as the *Rogers Report*, identified the failure of rubber O-rings sealing the joints in one of the boosters as the cause of the accident: "The specific failure was the destruction of the seals that are intended to prevent hot gases from leaking through the joint during the propellant burn of the rocket motor."[1] The external tank was destroyed, leading to the breakup of the orbiter.

Tragically, the possibility of an O-ring failure had been known for some time but was not properly communicated. Although the original cause of the disaster was a faulty design, the immediate cause—defective O-rings costing just a couple of dollars—lent its name to Michael Kremer's idea of an O-ring production function.[2] In contrast to the classical view of output as a deterministic function of some inputs, production is viewed as consisting of a wide range of independent subsystems all prone to failure and succeeding only if none of the subsystems fail.

The earliest example of a possible application in defense was the suggestion to interpret an aircraft carrier's flight deck operations as an O-ring production function.[3] That is, unless everything falls into place, catastrophic failure may result, as the USS Forrestal accident on 29 July 1967 sadly demonstrated. An example of an O-ring-like sequence, though not in name, is provided in the book Naval Operations Analysis. It states that for a submarine to succeed in destroying an enemy submarine, it would first have to detect it, then identify it as the correct target, work out a firing solution, launch the torpedo(es), at least one torpedo would have to make contact with the target, not become fooled by any decoys, and its exploder should eventually fire the warhead.[4] This sequence illustrates how every other kind of kill chain can also be interpreted as an O-ring production function as well, from the general idea of an OODA loop to the use of a drone strike to take out an individual terrorist.[5] It also holds for every individual weapon system, whether a WWII pursuit plane such as the P-40 Warhawk; an M1 Abrams battle tank; or last but not least, the F-35 Lightning II.

## The F-35: A State-of-the-Art O-Ring Production Function

Generally considered the most advanced fighter plane in existence, the F-35 not only displays extreme maneuverability and lethality but is a platform incorporating all the subsystems needed to conduct a strike against surface and aerial enemy targets alike. Still, it is an O-ring production function. Sticking with the OODA paradigm, a pilot unable to observe or orient would be unable to decide, let alone act. Thus, if any of an F-35's subsystems are incapacitated—either kinetically, by means of a cyberattack, or just by jamming—the whole platform is basically rendered useless. The mathematics behind the O-ring production function elucidates the dilemma.

The scenario assumes there are four tasks or subsystems needed to successfully complete a mission—such as "observe," "orient," "decide," and "act." The probabilities for these tasks to be successfully met are denoted by $p_1$, $p_2$, $p_3$, and $p_4$, respectively. The probability of mission success, assuming stochastic independence, is given by $p_1 \cdot p_2 \cdot p_3 \cdot p_4$, and the probability of mission failure by $1 - p_1 \cdot p_2 \cdot p_3 \cdot p_4$. To give a numerical example, even if every subsystem has a 90 percent chance of doing exactly what it is supposed to do, the mission success probability is $(0.9)^4 = 0.6561$; that is, the mission will fail in more than one out of three cases. If the subsystem success rate is increased to 95 percent, the probability of failure would go down to $1 - (0.95)^4 = 0.1855$, but the mission would still fail in almost one in every five cases. One would be mistaken, though, in assuming that increasing a subsystem's reliability is an easy way to alleviate the problem. Prima facie increasing (all) subsystems' reliabilities by 5 percentage points to increase the overall success probability by roughly 24 percent—from 0.6561 to 0.8145—looks a great idea. The cost of increasing any subsystem's reliability is exponential. It would cost less to increase its success probability from say 70 to 80 percent than increasing it from 80 to 90 percent, and the additional cost becomes ever more prohibitive the closer one gets to 100 percent. In terms of the O-ring production function theory and denoting the cost functions by $C_i(p_i)$, this reads as $C_i' > 0$ and $C_i'' > 0$. To illustrate the effect by using the simplest functional form for an O-ring-compatible cost function, $C_i(p_i) = 1/(1 - p_i)$, if a subsystem's reliability were to be raised from 70 to 80 percent, the cost would rise by 50 percent; raising reliability from 70 to90 percent would triple the cost. Finally, it should be superfluous to point out that a success probability equal to one is impossible to achieve—just as man is not perfect, there are no technologies available that never fail.

## From US (Not Only Air) Superiority to Anti-Access/Area Denial

Throughout history and up to and including WWII, warfare has largely been a numbers game. At the beginning of the Pacific War, the *Zero* was the most advanced fighter plane; Japan didn't have enough of them, though. In contrast to their American counterparts, Japanese pilots had combat experience, but again, there were too few. The German *Tiger* was considered the best tank of its time, vastly superior to say the American *Sherman*. Luckily for the Allies, though, there were many more *Shermans* around than *Tigers*.

All of this is in line with (tactical) warfare models. Bradley Fiske in 1905 and Frederick Lanchester in 1916 suggested that, in naval combat and aerial combat respectively, doubling a force's quantity should be more important than doubling its quality.[6]

From the end of WWII and through the Cold War decades, however, the picture changed as the US attained an ever-expanding gap in weapon technology advances over its peer rivals, Russia and China. The simple reason was economics. Just as a command economy could not compete with a free-market economy, neither could its defense industrial base. Russian numerical superiority did not help. The higher kill ratio of US weapon systems would have sufficed to halt Russian forces. Russian submarines could be tracked wherever they went, but not vice versa, and Russian commanders knew this. Precision bombing during the Vietnam War saw the advent of the "one bomb, one target" capability. US air superiority achieved its heyday during Operation Desert Storm. US stealth fighter-bombers could enter Iraqi airspace at will, and as Gen David Deptula noted in 2001, "The Gulf War began with more targets in one day's attack plan than the total number of targets hit by the entire Eighth Air Force in all of 1942 and 1943—more separate target air attacks in 24 hours than ever before in the history of warfare."[7]

The picture changed with 9/11 and the ensuing wars in Afghanistan and Iraq for three reasons. First, top-of-the-line air combat platforms were no longer considered necessary for counterinsurgency operations. Second, the cost of fighting two wars at the same time pushed back other expenditures, leading to a reduction in the numbers of F-22s and F-35s. Third, airspace was implicitly assumed to continue being uncontested. However, having had ample opportunities to study the American way of war over the decades US forces had reigned supreme, Russia and China—aware that they would remain unable to match US technological developments and military expenditure—chose to take an altogether different path. Rather than trying to play catch-up, they changed the game by embarking on doctrinal responses and strategies that would render US forces' superiority useless. The two countries would simply bar access to disputed areas, such as the Baltic Sea or the South China Sea respectively, and/or deny the ability to operate in those areas (i.e., A2/AD). In particular, by area denial US operations in the respective area would be impeded or slowed down at best, effectively preventing US forces to pursue the fundamental principle of tactical warfare which is, as the US Navy puts it, "Fire effectively first!"[8] Any attempt to enter the contested battlespace would be met by a both fierce and relatively cheap resistance. The cost of a Chinese DF-26 "carrier-killer" anti-ship missile comes at a fraction of any of its intended targets—it would make US losses unsustainable.

The outlook is bleak. War games keep proving that Chinese forces, by embarking on what Jeffrey Engstrom calls a "system confrontation" strategy and by conducting "system destruction warfare," would win against even the most advanced weapon systems, such as the F-35.[9] The basic elements of "system destruction" are attacking the joints, or nodes, by disrupting an adversary's flow; targeting networks and data links (thereby isolating his forces); targeting an adversary's high-value assets by disabling their essential elements (such as C2, ISR, and/or other essential subsystems); disabling an adversary's operational infrastructure; and slowing down an adversary's kill chains. To quote from the final report of the National Defense Strategy Commission:

> If the United States had to fight Russia in a Baltic contingency or China in a war over Taiwan . . . , Americans could face a decisive military defeat. These two nations possess precision-strike capabilities, integrated air defenses, cruise and ballistic missiles, advanced cyberwarfare and anti-satellite capabilities, significant air and naval forces, and nuclear weapons—a suite of advanced capabilities heretofore possessed only by the United States. The U.S. military would face daunting challenges in establishing air superiority or sea control and retaking territory lost early in a conflict. Against an enemy equipped with advanced anti-access/area denial capabilities, attrition of U.S. capital assets—ships, planes, tanks—could be enormous. The prolonged, deliberate buildup of overwhelming force in theater that has traditionally been the hallmark of American expeditionary warfare would be vastly more difficult and costly, if it were possible at all. Put bluntly, the U.S. military could lose the next state-versus-state war it fights.[10]

Cutting the number of US platforms—whether they are B2s, F-22s, or F-35s—certainly didn't help—nor does the fact that they are O-ring production functions.

## Mosaic Warfare

"Mosaic warfare" is a brainchild of the Defense Advanced Research Projects Agency (DARPA).[11] With the publications of the Mitchell Institute's research study authored by Gen David Deptula and Heather Penney[12] and a shortened version in Air Force Magazine,[13] the idea has entered military mainstream discussions.

The basic idea of mosaic warfare is amazingly straightforward and intuitively striking. If your adversary goes after your systems—"system destruction warfare"—just disaggregate your systems! Rather than putting all proverbial eggs (read subsystems or nodes) in one basket (read on board a single [O-ring production function] platform such as the F-35), use small platforms hosting disaggregated nodes instead. If your original force consisted of say four F-35s, opt for four small platforms hosting only one node each of the kill chain, say observation; opt for four small platforms hosting just another node of the kill chain, say orientation; and so on. And make sure every small platform can independently communicate with every other platform. If just one small platform were disabled, there would be no harm whatsoever because the remaining three platforms hosting the same subsystem or node would take over. In contrast, disabling one F-35's subsystem or node would render that F-35 ineffective. If every F-35 took just one hit, there would be no kill chain left. On the other hand, rendering a disaggregated kill chain network inoperable would require disabling not just any four small platforms but four identical platforms (i.e., all those hosting the same node). While the effect of this strategy is obvious—the probability of mission success should increase with mosaic warfare—its magnitude is not.

## Some Mosaic Warfare Mathematics

To illustrate the extent of the benefits to be expected when switching to mosaic warfare, consider an F-35's kill chain consisting of k nodes—using the OODA loop picture, k would equal four—and having an -ship formation. Assume that for the mission to be successful, it would suffice if just one ship gets through and delivers the kill. Then, using the same notation as in the F-35 section, the probability for an individual F-35 to get through would beand $p_1 \cdot p_2 \cdot \ldots \cdot p_k$ and the probability of failing or having to abort by, correspondingly, $1 - p_1 \cdot p_2 \cdot \ldots \cdot p_k$. With stochastic independence, the most likely scenario, the probability for all n ships to fail would be $(1 - p_1 \cdot p_2 \cdot \ldots \cdot p_k)^n$. Therefore, the probability of successfully completing a mission when using n F-35s (i.e., having at least one ship survive to deliver the kill) is

(1) $\text{prob (success|F-35s)} = 1 - (1 - p_1 \cdot p_2 \cdot \ldots \cdot p_k)^n$.

Alternatively, assume that instead of having all k nodes hosted by one (F-35) platform, k small sub-platforms are used for every F-35, each of which is responsible for just one of the k nodes. Then any of the k nodes would be compromised only if all its respectiven n sub-platforms are destroyed or rendered ineffective by other means. To isolate the mosaic warfare effect, all $p_1$ through $p_k$ are assumed to remain unchanged (most likely at least some of these probabilities would go up, as sub-platforms should be harder to detect due to being smaller; some sub-platforms could also be unmanned, increasing their maneuverability). Then, as the probability of node i to fail equals $(1 - p_i)^n$ the probability of node i surviving is $1 - (1 - p_i)^n$, and the probability of all nodes surviving and of mission success therefore is

(2) $\text{prob}(\text{success}|\text{mosaic warfare}) = (1 - (1 - p_1)^n) \cdot (1 - (1 - p_2)^n) \cdot \ldots \cdot (1 - (1 - p_k)^n)$.

The difference between (2) and (1) gives the increase in the chances of mission success due to switching to mosaic warfare.

To visualize the magnitude of the influence of mosaic warfare, assume that all $p_i$ are identical, henceforth denoted by $p := p_1 = p_2 = \cdots = p_k$.[14] Then (1) and (2), respectively, can be simplified to

(1a) $\text{prob}(\text{success}|\text{F-35s}) = 1 - (1 - p^k)^n$ and (2) becomes

(2a) $\text{prob}(\text{success}|\text{mosaic warfare}) = (1 - (1 - p)^n)^k$.

This formula allows for evaluating the outcome of different scenarios by means of a simple pocket calculator.

It is obvious that for any one-ship mission there cannot be a mosaic warfare effect. Therefore, assume $n = 2$ (i.e., a two-ship mission) and $k = 4$ (OODA). With $p = 0.9$, (1a) yields 0.88173279, while (2a) yields 0.96059601 (i.e., switching to mosaic warfare would improve the chances of mission success by about 7.9 percentage points). However, as an F-35 mission success probability of around 88 percent still sounds pretty good and is not exactly in line with "the U.S. military could lose the next state-versus-state war it fights",[15] try $p = 0.7$ pinstead. (1a) would yield 0.42255199 – now the mission would fail more often than not – while (2a) would yield 0.68574961, i.e., Mosaic Warfare would increase the chance of winning by about 26.3 percentage points and raise it above the two-out-of-three level.[16]

Formulae (1a) and (2a) can be used to easily evaluate the outcomes of other scenarios by toying with k, n, and p (i.e., whether it is a change in the number of subsystems or nodes, the number of platforms, or the reliability of the subsystems). The results stay true: mosaic warfare will always improve the chances of mission success, and the more even the chances of a successful F-35 mission, the higher the benefits to be gained.

## Summary

This article was never intended to prove the validity of the mosaic warfare concept. Particularly, it did not even try to address technological or doctrinal questions such as the danger of communications between sub-platforms being compromised (mission failure would be obvious; on the other hand, should an F-35 become isolated, it could still try to proceed). Neither did it address how long it would take to develop sub-platforms and bring them into service (the South China Sea conflict could turn hot any time soon); the time it takes to devise a new doctrine (as long as the commander in the field remains unconvinced, all is in vain); or the compatibility of "traditional" air war (i.e., putting one's trust in highly sophisticated but more vulnerable O-ring production function weapon systems) and applying mosaic warfare (can they be run in parallel?).

That said, for any new idea to live on, the word must get out, the story, including every single facet, has to be circulated. This article concentrates on the likely magnitude of the mosaic warfare effect on mission success. Using a not-exactly-rocket-science mathematical argument, the article suggests that this approach can, more often than not, substantially improve the chances of mission success in scenarios where traditional approaches are bound to fail. Considering that mosaic warfare systems can come a lot cheaper than the single-platform weapon systems in use today, mosaic warfare could begin to look ever more attractive. ✪

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**Editor's Note**

This article originated with and is co-published with our fellow Air University Press journal, the *US Air Force Journal of the Americas* (*JotA*), which will also publish the article in Spanish and Portuguese. Be sure to visit *JotA* at https://www.airuniversity.af.edu/JOTA/.

## Notes

1. "Report of the Presidential Commission on the Space Shuttle Challenger Accident (In compliance with Executive Order 12546 of February 3, 1986):" 40 https://history.nasa.gov/rogersrep/v1ch4.htm.

2. Michael Kremer, "The O-Ring Theory of Economic Development," *Quarterly Journal of Economics* 108, no. 3 (August 1993): 551–75.

3. Gene Rochlin, Todd La Porte, and Karlene Roberts, "The Self-Designing High-Reliability Organization: Aircraft Carrier Flight Operations at Sea", *Naval War College Review* 40, no. 4 (1987): 76–90.

4. Daniel H. Wagner, W. Charles Mylander, and Thomas J. Sanders, eds., Naval Operations Analysis (Annapolis, MD: Naval Institute Press, 1999).

5. The concept's very essence carries over to other nonstochastic models, such as Driver and DeFeyter's, where "intelligence", "resources," and "political opportunity structures" are "multiplied, as opposed to summed, to reflect that all components are necessary" to have a chance of winning in unconventional warfare. William "Dave" Driver and Bruce E. DeFeyter, *The Theory of Unconventional Warfare: Win, Lose, and Draw (*Monterey, CA: Naval Postgraduate School, 2008).

6. Bradley A. Fiske, American Naval Policy, Proceedings of the United States Institute 31 (January 1905): 1-80; Frederick W. Lanchester, Aircraft in Warfare: The Dawn of the Fourth Arm (London: Constable, 1916). While Lanchester's model was about aerial combat, unbeknown to him a paper written by Jehu Chase in 1902 as a lieutenant at the Naval War College was a forerunner in describing naval warfare. The mathematics were the same, but Chase, in contrast to both Lanchester and Fiske, had even taken staying power, i.e., defensive characteristics, into account. Using his model, Chase in particular pointed out the advantages of the tactics of isolating enemy forces. It was this recommendation that immediately led to the paper being classified. It was not declassified until 1972. See Jehu V. Chase, "*A Mathematical Investigation of the Effect and Superiority of Force in Combats upon the Sea*" (unpublished paper, Naval War College Archives, Newport, RI, RG 8, Box 109, XTAV [1902]).

7. David A. Deptula, Effects-Based Operations: Change in the Nature of Warfare (Arlington, VA: Aerospace Education Foundation, 2001).

8. See Wayne P. Hughes, *Fleet Tactics and Coastal Combat*, 2nd ed. (Annapolis, MD: Naval Institute Press, 2000).

9. Jeffrey Engstrom, *Systems Confrontation and System Destruction Warfare: How the Chinese People's Liberation Army Seeks to Wage Modern Warfare* (Santa Monica, CA: The RAND Corporation, 2018).

10. Eric Edelman and Gary Roughead, Providing for the Common Defense: The Assessment and Recommendations of the National Defense Strategy Commission (Washington, DC: United States Institute of Peace, 2018): 14.

11. DARPA, Strategic Technology Office Outlines Vision for Mosaic Warfare, (DARPA, August 4, 2017, https://www.darpa.mil/news-events/2017-08-04).

12. David Deptula and Heather Penney, Restoring America's Military Competitiveness: Mosaic Warfare (Arlington VA: The Mitchell Institute for Aerospace Studies, 2019).

13. David Deptula and Heather Penney, "Mosaic Warfare", *Air Force Magazine* 102, no. 11 (2019): 51–55.

14. If the sub-systems' cost functions are identical, $p_1 = p_2 = \cdots = p_k$ would be the cost minimizing/success maximizing solution anyway.

15. Edelman and Roughead, *Providing for the Common Defense.* See previously cited quote in the section on anti-access/area denial.

16. It should be noted, though, that if  was reduced further, the gain, while always being positive, will eventually become smaller again.