Bipolar Coding and Baseband Recovery

BY STATUTORILY EXEMPT

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It is sometimes desirable to code a digital baseband (random binary) signal in a three-level digital representation. For example, direct recording of a binary waveform on conventional longitudinal head tape machines is unacceptable because of the poor low frequency response of these recorders below 200 cps and the predominance of low frequency energy in random binary data. Bipolar coding, as will be shown, removes the requirements for excellent low frequency response.

Bipolar coding is achieved when every other "1" in a binary sequence of "1's" and "0's" becomes a "−1". The coding rule is illustrated in Fig. 1.

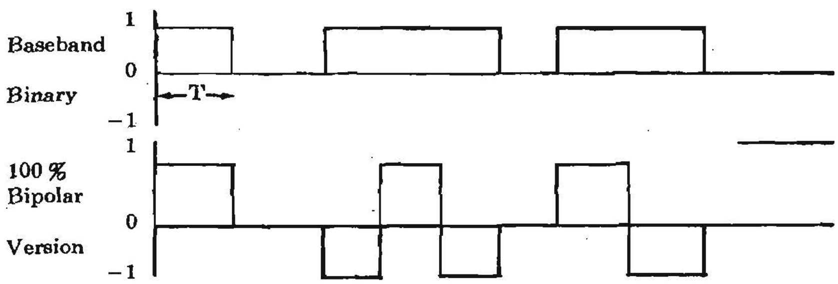

Fig. 1

A simple electronic bipolar coder is shown in Fig. 2.

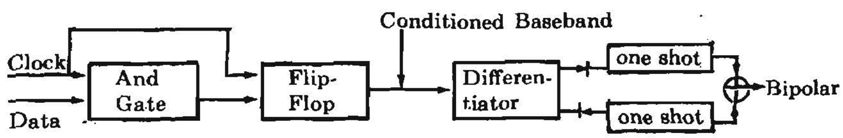

Fig. 2

Perhaps the simplest way of arriving at the power spectrum of the binary waveform is to analyze the system in Fig. 2; in doing this some very interesting features of the system will be brought out.

The output of the flip-flop is referred to as conditioned baseband where "1's" in the original binary data show up as transitions in the conditioned binary sequence. It is of interest to obtain both the autocorrelation function and the power spectrum of the conditioned binary sequence before obtaining the power spectrum of the bipolar signal. For convenience, let the conditioned binary signal take on values of A or $-A$. The autocorrelation function for the conditioned baseband $\phi_{CB}(\tau)$ may be obtained a number of ways; for example, by treating the conditioned signal as a Markov process or by a simple binomial summation. The latter method results in

$$\phi_{CB}(\tau) = \phi_{CB}(nT) = A^2 \sum_{k\ \text{even}}^{n} \frac{n!\, p^k(1-p)^{n-k}}{k!(n-k)!} - A^2 \sum_{k\ \text{odd}}^{n} \frac{n!\, p^k(1-p)^{n-k}}{k!(n-k)!} \tag{1}$$

$$\phi_{CB}(nT) = A^2(1-2p)^n$$

where p is the probability of a "1" occurring in the original binary data and T is the time interval of one bit in the original binary stream. $\phi_{CB}(\tau)$ is illustrated in Fig. 3.

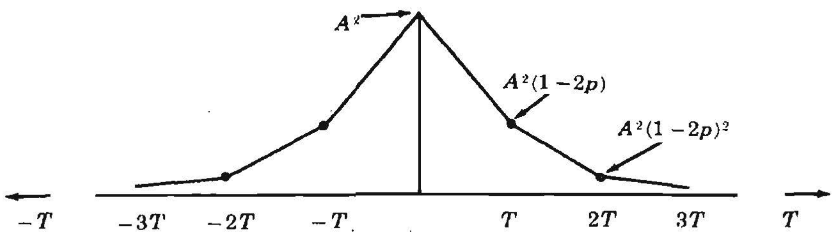

Fig. 3

The autocorrelation function reflects the bias in the original data through the transition probability p. The power spectrum associated with $\phi_{CB}(\tau)$ is found by solving the Wiener-Kinchine integral relationship. (See Appendix A.)

$$\Phi_{CB}(w) = \int_{-\infty}^{\infty} \phi_{CB}(\tau)\, e^{-jw\tau} d\tau = \frac{4A^2}{w^2T} \frac{p(1-p)(1-\cos wT)}{2p^2+(1-2p)(1-\cos wT)} . \tag{2}$$

The above correlation function and power spectrum represents an excellent approximation for the output of a conventional delta modulation coder whose input is speech when $(1/20) < p < (1/6)$. The reader is encouraged to plot Equation 2 as a function of w for various p to observe the behavior of $\Phi_{CB}(w)$.

Proceeding further in the analysis of the bipolar coder, one would like to know the power spectrum after the differentiator. Since

$$\phi_{CB}''(\tau) = \overline{f'(t)\, f'(t+\tau)} = \frac{-1}{2\pi}\int_{-\infty}^{\infty} w^2\, \Phi_{CB}(w)\, e^{jw\tau}\, dw, \qquad (3)$$

it follows immediately that the power spectrum after the differentiator is simply

$$\Phi_{\text{diff}}(w) = \frac{4A^2}{T}\ \frac{p\,(1-p)(1-\cos wT)}{2p^2 + (1-2p)(1-\cos wT)}\ . \qquad (4)$$

The one shots may be regarded as a linear network having an impulse response $h(t) = B,\ 0 < t < \Delta,\ \Delta \leq T.$ The power spectrum associated with the bipolar coded signal can now be obtained with the aid of the relationship between the input and output power spectra of a linear network.

$$\Phi_{\text{out}}(w)^2 = \Phi_{\text{in}}(w)\, |H(w)|^2 \text{ where } H(w) = \int_{-\infty}^{\infty} h(t)\, e^{-jwt} dt$$

$$\Phi_{\text{Bipolar}}(w) = \frac{B^2\Delta^2 \sin^2 (w\Delta)/2}{((w\Delta)/2)^2}\left[\frac{p\,(1-p)(1-\cos wT)}{2p^2 + (1-2p)(1-\cos wT)}\right]. \qquad (5)$$

Equation 5 is illustrated in Fig. 4 for $\Delta = T$ (100% bipolar) $p = 1/2$. The reader again can evaluate Equation 5 for other Δ's and p's.

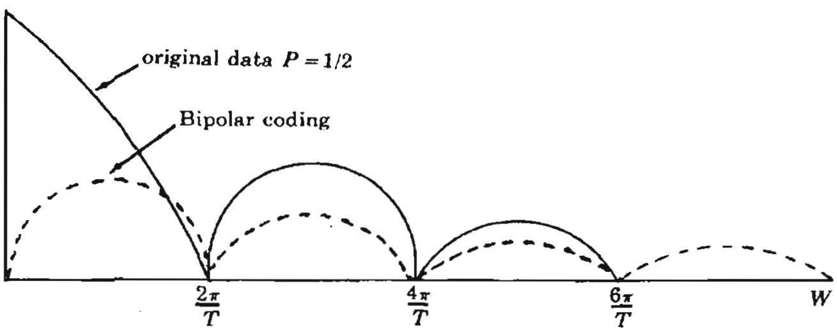

Fig. 4

It is evident that the predominance of low frequency power does not exist for the bipolar coded version of the data for $\Delta = T\ p = 1/2$.

The recovery of the original baseband data from the bipolar coded version, imbedded in gaussian noise, is quite simple. For a 100% bipolar signal, a full wave rectifier, slicer, and sampler are all that are required to perform the recovery. The probability of error P_ϵ incurred in the recovery will not be evaluated and compared to the optimum recovery of binary data in noise. The probability of error is readily determined from the probability densities functions (p.d.f.) shown in Fig. 5.

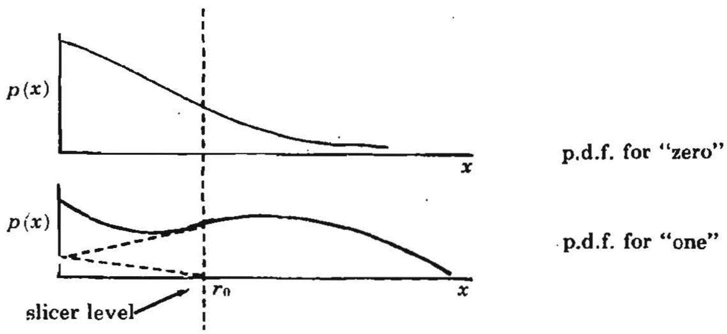

Fig. 5

It is evident that P_ϵ (for $p = 1/2$) is given by:

$$P_\epsilon = 1/2 \left[2 \int_{r_0}^{\infty} \frac{e^{-(x^2/2\sigma^2)}}{\sqrt{2\pi\sigma^2}}\, dx + \int_{-r_0}^{r_0} \frac{e^{-(x-B)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\, dx \right]. \qquad (6)$$

In order to minimize P_ϵ an optimum threshold $\hat{r}_0$ must be determined. The minimum occurs for $\partial P_\epsilon / \partial r_0 = 0$.

Thus,

$$\frac{\partial P_\epsilon}{\partial r_0} = f_1(\infty) \frac{\partial(\infty)}{\partial r} - f_1(r_0) \frac{\partial r_0}{\partial r_0} + f_2(r_0) \frac{\partial r_0}{\partial r_0} - f_2(-r_0) \frac{\partial(-r_0)}{\partial r_0} \qquad (7)$$

where

$$f_1(\ ) = \frac{2e^{-(x^2/2\sigma^2)}}{\sqrt{2\pi\sigma^2}} \qquad f_2(\ ) = \frac{e^{-(x-B)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}};$$

after rearranging terms, $\hat{r}_0$ becomes

$$\hat{r}_0 = (\sigma^2/B) \cos h^{-1} \left[e^{B^2/2\sigma^2} \right].$$

Defining $B^2/2\sigma^2$ as the average signal-to-noise ratio (S/N),

$$\hat{r}_0 = \frac{\sigma}{\sqrt{2}\sqrt{S/N}} \cos h^{-1} \left[e^{S/N} \right] . \qquad (8)$$

Equation 8 describes how r_0 must change with S/N in order to minimize P_ϵ. When $\hat{r}_0$ is used in Equation 6, the minimum error rate results. Figure 6 compares bipolar recovery with optimum binary recovery for $p = 1/2$.

SUMMARY

The correlation function and power spectrum for a conditioned baseband signal have been obtained. From that exercise, the power spectrum of a bipolar coded signal followed. The latter spectrum reflects the bias of the original data and the effect of the "on" time duration Δ.

The recovery of the original data from the bipolar coded version has been investigated, and the optimum threshold determined and compared against the standard baseband on a P_ϵ basis.

APPENDIX A

Derivation of the Power Spectrum of a Conditioned Baseband Signal

It was shown in the body of the paper that the autocorrelation function of a conditioned baseband binary signal is of the form

$$\Phi_{CB}(\tau) = (1-2p)^{n} .$$

Thus the power spectrum is given by

$$\Phi_{CB}(w) = \Phi^{+}(w) + \Phi^{-}(w)$$

where

$$\Phi^{+}(w) = \int_{0}^{\infty} \phi_{CB}(\tau)\, e^{-jw\tau} d\tau .$$

Let

$$\Phi_{1}^{+}(w) = \int_{0}^{T} \left(A^{2} - [A^{2} - (1-2p)\, A^{2}]\, \frac{\tau}{T}\right) e^{-jw\tau} d\tau$$

$$= (A^{2}/jw)\, [1 - (1-2p)\, e^{-jwT}] + (2pA^{2}/w^{2}T)\, [1-e^{-jwT}] ;$$

$$\therefore \quad \Phi_{2}^{+}(w) = \Phi_{1}^{+}(w)\, (1-2p)\, e^{-jwT} ,$$

and

$$\Phi_{n}^{+}(w) = \Phi_{1}^{+}(w)\, (1-2p)^{n-1}\, e^{-jw(n-1)T} ;$$

$$\therefore \quad \Phi^{+}(w) = \Phi_{1}^{+}(w) \sum_{r=0}^{\infty} (1-2p)^{r}\, e^{-jwrT} ,$$

and

$$\Phi_{CB}(w) = \Phi^{+}(w) + \Phi^{-}(w)$$

$$= \frac{2A^{2}p}{w^{2}T} \left[\frac{1-e^{-jwT}}{1-(1-2p)\, e^{-jwT}} + \frac{1-e^{jwT}}{1-(1-2p)\, e^{jwT}}\right]$$

$$= \frac{4A^{2}}{w^{2}T}\; \frac{p\,(1-p)(1-\cos wT)}{2p^{2} + (1-2p)(1-\cos wT)} .$$

References:

Wolf, *P.G.C.T.*, Vol. 14, Feb. 66.

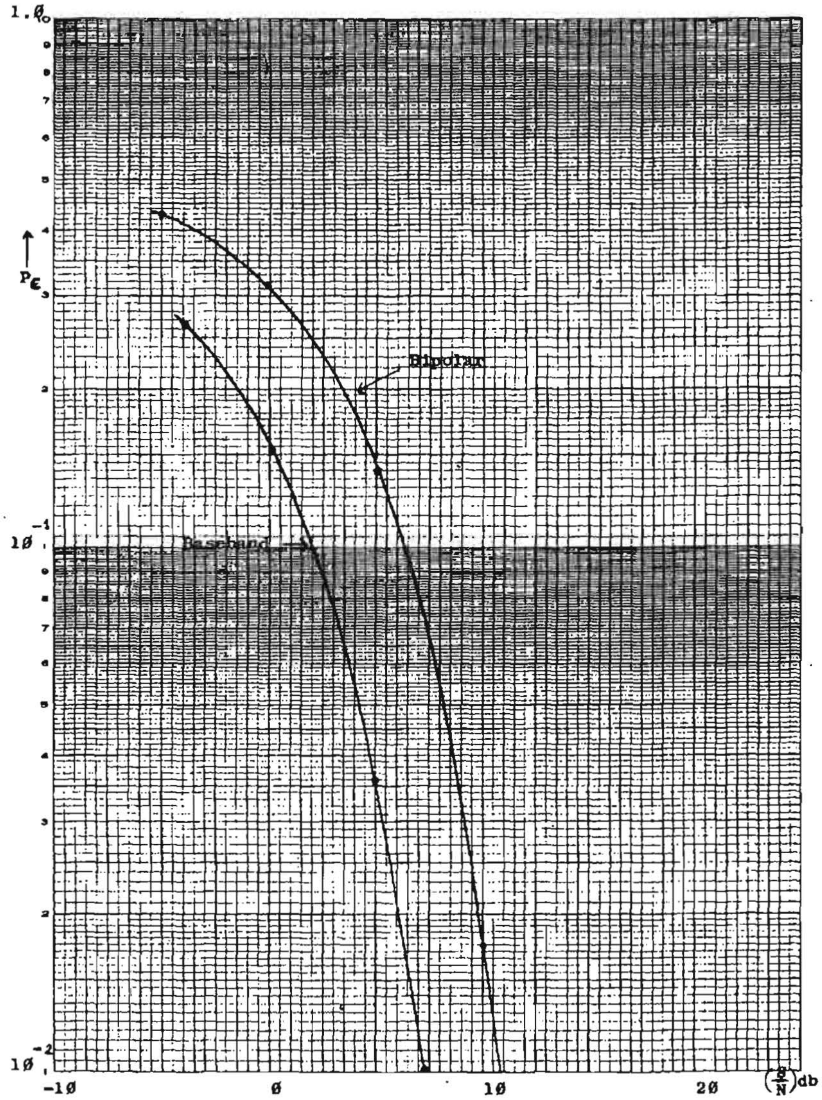

Fig. 6