Why Analog Computation?

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An introduction to analog computation containing a brief description of the analog computer and problems in which it can be advantageously applied. Both analog computers and systems combining analog and digital techniques are discussed in order to show why the Agency's interest in this computation area has increased.

Why analog computation? With the interest in analog computing equipment rapidly increasing in our digitally oriented Agency, this is a question many of us must ask. The preponderence of digital computing equipment in this Agency would preclude analog computation from consideration if the two types of computers performed the same operations equally well; but this is not the case. A comparison of digital and analog computer applications reveals a basic difference in their operation. The digital computer performs numerical operations on discrete signals; in contrast, the analog computer performs algebraic and integro-differential operations upon continuous signals. Therefore certain operations, which are difficult to program on a digital computer, are available inherently on the analog machine. In order to appreciate where an analog computer can be advantageously applied, one must become more familiar with what it is and how it is used.

Before discussing problem areas in which the analog computer possesses an advantage, let us briefly consider the fundamentals of its operation.

The heart of the computer is the high-gain D.C. amplifier—either vacuum tube or transistor—that, when properly connected with passive components, forms the basic operational element. The schematic representation for an operational amplifier is shown in Fig. 1.

If the passive components in both feedback and input arms are entirely resistive, the circuit of Fig. 1 adds the applied voltages in proportion to the ratios of the individual resistors; while if the feedback impedance is capacitive, the circuit integrates the sum of the applied voltages. The schematic diagrams for an amplifier used as a summer (it is called an inverter if it has only one input) and as an integrator are shown in Fig. 2.

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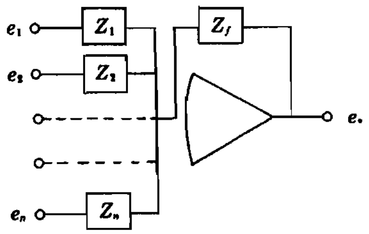
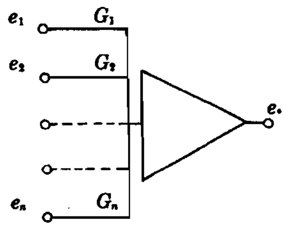


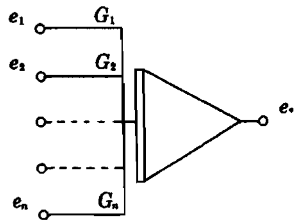
$$e_o = -\left(\frac{Z_f}{Z_1}e_1 + \frac{Z_f}{Z_2}e_2 + \cdots + \frac{Z_f}{Z_n}e_n\right)$$

Fig. 1.—**Operational Amplifier.**



$$e_o = -(G_1e_1 + G_2e_2 + \cdots + G_ne_n)$$

Adder



$$e_o = -\int_0^t (G_1e_1 + G_2e_2 + \cdots + G_ne_n)dt$$

Integrator

Fig. 2.

If the simple input and feedback impedances are replaced with complex networks, either passive or active, the amplifier circuit will develop more complicated transfer functions than those shown in Fig. 2. In addition to the basic amplifiers, the general-purpose analog computer usually contains a variety of special purpose units; for example, multipliers to form the product of two or more variables, fixed and variable-diode function generators to perform various nonlinear operations on the variables, switches to start and modify the operations, and comparators to make elementary decisions based on the value of a particular variable. It is the compatability and simplicity of interconnection of these various components that give the analog computer its flexibility and versatility.

With this brief discussion of the analog computer itself as background, the solution of problems with it will be considered. The analog computer has basically two modes of operation. The first mode is a simulation of the mathematical equations that describe a system; while the second is a simulation of the functions that a system must perform in processing continuous signals "on line". The first mode is used when the basic parameters of the system are to be investigated; the second, when its total characteristics must be evaluated. The choice of the simulation technique to be used for a particular problem usually depends on the nature of the solution required. Either mode of simulation or a combination of some features of both may yield the most fruitful results depending upon the degree of interest in the detailed operation of the specific parts and the overall operation of the entire system. For simulation on the analog computer, it is not necessary that a problem be electrical in nature since the solutions are obtained from an analogy between the physical variables—be they electrical, mechanical, mathematical or the like—and the computer voltages. After a problem has been simulated, care must always be exercised in checking trial solutions against experimental or analytical data to insure that the solutions do satisfy, at least at some particular points, the original statement of the problem.

Although a detailed simulation obtained from the defining equations offers many advantages in the analysis of the operation of a system, only a few of the more general ones will be discussed here. The first advantage is that the individual parameters may be isolated on the computer so that each may be varied independently; and, therefore, the required response function may be optimized systematically. This mode of operation has particular appeal for the electrical engineer since the machine simulation may be used in the same manner as the "bread board" circuits to which he is accustomed, but with increased flexibility and more rapid and simpler modification of

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the parameters than with the actual circuit. A second advantage is that, since ideal elements can be substituted in the simulation for the physical components, information can be obtained about the parameters that can not be gotten by direct measurement on an actual circuit. Still another advantage is that the solution of non-linear problems is only slightly more difficult than the solution of linear ones; in fact, a non-linear problem is usually programmed as a linear one, and then the non-linear function is inserted in place of its linear approximation. In this manner the non-linear problems, that resist analysis in all but the simplest cases, can be solved quite readily on the analog computer with little additional effort.

To illustrate this mode of operation, the following problem of particular current interest to the agency is discussed. Various tunnel diode circuits are to be evaluated in order to determine those which look most promising for use in high speed digital computers and similar applications; and then these circuits are to be investigated in greater detail to develop design criteria. To analyze each proposed circuit on the analog computer, a circuit diagram is drawn using a linear model for the tunnel diode; and from this circuit are written the system equations. Although the analog computer does not operate at the same speeds and voltage levels as the tunnel diode circuits, it can be made to represent their operation by the proper time and amplitude scaling of the equations. After the linear equations are programmed for solution on the computer, the negative resistance characteristic of the diode, as shown in Fig. 3, may be set up on a variable-diode function generator and inserted in the program in place of the linear resistance.

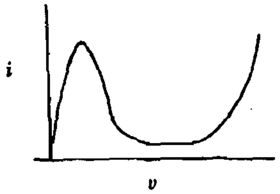

Fig. 3.—Tunnel Diode Characteristic.

The most direct application for this non-linear simulation is the investigation of the effect on the circuit response caused by varying the linear components, with the diode characteristic held constant, because these components can rapidly be modified by changing with potentiometers the gain of the amplifiers that represent them. In addition, the diode characteristic itself may be easily modified to conform to any specified values that correspond to a particular diode's parameters, or to parameter values that are not now available in actual diodes but that appear to offer some potential advantage in the circuit. This brief discussion illustrates some of the advantages that the analog computer possesses in the solution of this class of problems.

The simulation of the functions that a system must perform in processing continuous or, as they are sometimes called, analog signals is also readily accomplished on the analog computer. A simple example of an analog system is a radio receiver since it must filter, amplify and demodulate continuously incoming signals. All these operations can be simulated on a general purpose analog computer by interconnecting the components available. For the processing of limited bandwidth signals such as speech, the computer is capable of operating directly upon the signal, or a tape recorded copy of it; but for wider bandwidth signals, such as those encountered in a radio receiver, it is necessary to expand the time scale of the simulation and operate upon slowed down or simulated signals. The advantage of simulating a system, either in real or expanded time, on the analog computer is that instead of constructing a special-purpose equipment just to determine the efficacy of a proposed scheme in processing some particular data, the standard components on the computer can be rapidly connected and tested. If the simulation reveals that the process justifies the construction of a special-purpose machine, the simulation can then be used to compile information about the various system parameters that can be used to simplify the design of the final machine.

To illustrate this type of simulation the following example on correlation is presented. Since the correlation function is a measure of the similarity of two signals, in many instances it would be advantageous to accomplish this comparison "on line", as rapidly as possible, so that the resulting information may be used to make an immediate decision. In order to avoid a lengthy discussion of the correlation process itself, the problem will be stated simply as the determination of the correlation function defined by the following equation:

$$R_{12}(\tau) = 1/T\int_0^t X_1(t)\, X_2(t+\tau)\, dt$$

If X_1 and X_2 are the same signal, this is called an autocorrelation function; and if they are different signals, it is a crosscorrelation function. This equation indicates that a product must be formed of the two signals at various offsets in time (τ) and that each product must then be integrated over the specified interval to determine the

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correlation function. For solution on a digital computer it would be necessary to sample and store the two waveforms, multiply the corresponding samples for each offset, and then numerically integrate all the resulting products. Since a very large number of samples is necessary to accurately represent most signals, even the most modern digital computers would have difficulty in performing all the required operations rapidly enough to make the decisions "on line." In contrast, analog equipment—either optical or electronic—which includes delay lines can instantaneously multiply the two signals as received and integrate the product for the various offset signals obtained from the delay line. The analog computer can prove extremely useful in evaluating the effectiveness of correlation in producing the required information because it contains the multipliers and integrators needed for testing the process, and delay lines can be simulated when an expanded time scale is used. Although the limited bandwidth of the operational amplifiers restricts the class of signals that can be correlated directly on the analog computer, some analysis of speech is within the capabilities of present analog computers. Although higher frequency signals can not be correlated directly on the analog computer, they can be investigated either by reducing their speed on a tape recorder or using simulated waveforms in place of the actual signal.

Now that the application of an analog computer to typical problems has been discussed, let us again compare it to a digital computer. The analog computer readily solves problems with a single dependent variable. Usually this variable is time, but others can be programmed. In contrast, since the operation of the digital computer is unaffected by the number of independent variables in the problem, it can be programmed with a greater degree of flexibility than the analog computer when more than one independent variable is present in a problem. The cost of increasing the accuracy with which the digital computer calculates a particular value is only time, while the analog computer would have to be reconstructed of more exact components to increase its accuracy. Therefore, if extreme accuracy is needed at each point, the digital computer is the best device; but remember that it provides no information between these points, while the analog computer, though less exact, has a continuous solution. The fundamental differences in the two types of computers have led to machines that combine some features of each in a hybrid system.

The oldest of these combined machines is the Digital Differential Analyzer. This machine is actually a special-purpose digital computer that is programmed like an analog computer and not with a stored set of instructions. In this machine a group of numerical integrators replaces the operational amplifiers of the analog computer as the basic computing element. Since these numerical integrators can be connected in parallel on the Digital Differential Analyzer, it can solve a set of equations more rapidly than a general-purpose digital computer, although still not as quickly as an analog computer. With this parallel operation, it is possible to obtain solutions more rapidly, while still taking advantage of the digital computer's greater accuracy. A more recent development has been the iterative analog computer which incorporates digital circuitry to provide logic and storage. Iterative computers may be programmed to operate faster than real time and obtain the solution repetitively; electronic switches are then employed to store the results of one solution and use these results as initial conditions for the subsequent solution so that the problem may be solved iteratively. The inclusion of logical control of the switches enables the programmer to use several different solution rates in a single problem and thereby handle problems of more than one variable. The compressed time scale—faster than real time—permits the simulation of systems in which the uncertainties are known only in a probabilistic sense and therefore require statistical analysis. The compressed time scale allows an adequate number of samples to be taken in a reasonable length of time so that meaningful statistics are obtained. A third approach to combining the best features of each type of computer has been to design circuitry which couples two standard computers. This additional equipment permits communication between the two computers by providing intermediate storage and conversion between the analog and digital variables of the individual machines. This approach offers the greatest flexibility in programming at the cost of data conversions that are not required in the special-purpose machines previously discussed. Whether to use a hybrid system or one of the basic machines depends, of course, on the nature and complexity of the individual problem.

It is hoped that this paper has provided some insight into why the Agency's interest in analog computation has increased. Since analog computation possesses advantages in some problem areas, it is necessary to investigate both analog computers and hybrid systems in order to provide the most efficient means of computation for Agency problems.